RANDOMPROCESSES

1. Define Stochastic process.
2. Classify Random Process.
3. What is a continuous random sequence? Give an example.
4. Give an example of stationary random process and justify your claim.
5. Distinguish between wide sense stationary and strict sense stationary processes.
6. Prove that the first order stationary process has a constant mean.
7. What is a Markov processes.
8. Define Markov chain and one – step transition probability
9. Give an example of Markov Processes.
10. Find the invariant probabilities for Markov chain $\{X_n; n \geq 1\}$ with state space S = $[0,1]$ and one-step TPM P = $\begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{bmatrix}$.
11. What is stochastic matrix? When is said to be regular?
12. Define irreducible Markov chain and state Chapman – Kolmorgow theorem.
13. What is meant by Steady state distribution of Markov chain?
14. State the Postulates of Poisson process.
15. State any two properties of Poisson process.
16. What will be the super position of n independent Poisson processes with respective average rates $\lambda_1, \lambda_2, \ldots \lambda_n$?
17. Define auto correlation function and state any two of its properties.
18. Define autocorrelation function and prove that for a WSS process $R_{XX}(-\tau) = R_{XX}(\tau)$.
19. Define cross correlation function and state any two of its properties.
20. Given the autocorrelation function for a stationary ergodic process with no periodic components is R(τ) = $25 + \frac{4}{1+6\tau^2}$. Find the mean and variance of the process.
21. If the autocorrelation function of a stationary process is $R_{XX}(\tau) = 36 + \frac{4}{1+3\tau^2}$, find the mean and variance of the process.
22. Define birth and death process.

PART- B

1. Define a random (stochastic) processes. Explain the classification of random process. Give an example to each class.
2. Consider a random process y(t) = x(t) cos(ω_0t + θ) where x(t) is distributed uniformly in (-π,π) and ω_0 is a constant. Prove that y(t) is wide – sense stationary.
3. Two random processes X(t) and Y(t) are defined by X(t) = Acosωt + Bsinωt and Y(t) = Bcosωt - Asinωt. Show that X(t) and Y(t) are jointly Wide-Sense stationary if a and B are uncorrelated random variables with zero means and same variances and ω is constant.
4. Show that the process X(t) = A cosλt + B sinλt where A and B are random variables is wide sense stationary if (i) E(A) = E(B) = 0 (ii) $E(A^2) = E(B^2)$ and (iii) E(AB) = 0.
5. Show that the random process X(t) = A sin(ωt+φ) where φ is a random variable uniformly distributed in (0,2π) is (i) first order stationary (ii) stationary in the wide sense.
6. For a random process X(t) = Y sinωt, Y is an uniform random variable in (-1, 1). Check whether the process is wide sense stationary or not.

7. If X(t) is a wide sense stationary process with autocorrelation function $R_{XX}(\tau)$ and if Y(t) = X(t+α) – X(t-α) , show that $R_{YY}(\tau) = 2R_{XX}(T+2\alpha) - R_{XX}(T-2\alpha)$.
8. If X(t) = 5 sin(ωt+φ) and Y(t) = 2cos(ωt+φ) where ω is a constant θ+φ = π/2 and is a random variable uniformly distributed in (0,2π), find $R_{XX}(\tau)$, $R_{YY}(\tau)$, $R_{XY}(\tau)$ and $R_{YX}(\tau)$. Verify two properties of autocorrelation function and cross correlation function.
9. If the process $\{N(t); t \geq 0\}$ is a Poisson process with parameter λt, obtain $P[N(t) = n]$ and $E[N(t)]$.
10. Find the mean and autocorrelation of of Poisson process.
11. State the Postulates of Poisson process. Discuss any two properties of Poisson processes.
12. Prove that the sum of two independent Poisson process is also a Poisson Process.
13. Let X be a random variable which gives the interval between two successive occurrences of a Poisson process with parameter λ. Find out the distribution of X.
14. Given a random variable Y with characteristic function $\varphi(\omega) = E(e^{i\omega y})$ and a random process defined by X(t) = cos(λt + y). Show that $\{X(t)\}$ is stationary in the wide sense if φ(1) = φ(2) = 0.
15. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is likely to drive again he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a '6' appeared. Find (i)the probability that he takes train on the third day (ii) the probability that he drives to work in the long run.
16. Three boys A, B and C are throwing a ball to each other. 'A' always throws the ball to 'B' and 'B' always throws the ball to 'C' but 'C' is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.
17. The transition probability matrix of a Markov Chain $\{X_n\}$, n = 1, 2, 3, … having 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.5 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is P(0) = (0.7, 0.2, 0.1) . Find (i) $P(X_2 = 3)$ (ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.
18. The one step TPM of a Markov chain $\{X_n; n = 0,1,2\}$ having state space S = $[1,2,3]$ is $P = \begin{bmatrix} 0.1 & 0.5 & 0.5 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is P(0) = (0.7, 0.2, 0.1) . Find (i) $P(X_2 = 3)$ (ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 0)$ $P(X_2 = 3/ X_0 = 1)$.
19. Let $\{X_n\}$, n = 1, 2, 3, … be a Markov chain with state space S = 0,1,2 and one step Transition Probability Matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1 & 0 \end{bmatrix}$

   (i) Is the chain ergodic? (ii) Find the invariant probabilities.

20. Show that random process X(t) = A cos(ωt+θ) is wide sense stationary if a and ω are constants and θ is uniformly distributed random variable in (0,2π).
21. For a random process X(t) = Y , Y is uniformly distributed random variable in (-1, 1). Check whether the process is wide sense stationary or not.

22. The process $P\{X(t)=n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, n=1,2,... \\ \dfrac{at}{1+at}, n=0 \end{cases}$

Show that X(t) is stationary.

23. If X(t) = A cosλy + B sinλt ; t ≥ 0 is a random process where A abd B are independent N(0, σ^2) random variables, examine the stationary of X(t).

24. Let X(t) = A cos(ωt+Y) be a random process where Y and ω are independent random variables. Further the characteristic function φ of Y satisfies φ(1) = 0 and φ(2) = 0 , while the density function f(ω) of ω satisfies f(ω) = f(-ω). Show that X(t) is wide sense stationary .

25. The autocorrelation function of a stationary random process is $R_{XX}(\tau)$ = $16 + \dfrac{9}{1+6\tau^2}$. Find the variance of the process.

26. Let $\{X(t); t \geq 0\}$ be a random process where

X(t) = total number of points in the interval (0,t) = k , say

$= \begin{cases} 1 \text{ if } k \text{ is even} \\ -1 \text{ if } k \text{ is odd} \end{cases}$ Find the autocorrelation function of X(t).