## UNIT – IV TWO DIMENTIONAL RANDOM VARIABLES
## PART – A

1. If X and Y are random variables having density function f(x,y) = $\frac{1}{8}(6-x-y)$, $0 < x < 2$, $2 < y < 4$, Find P( X + Y < 3 ).
2. State the equation of two regression lines.what is the angle between them?
3. The following table gives the joint probability distribution of X and Y. Find (i) marginal density function of X. (ii) marginal density function of Y.

| Y X | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 0.1 | 0.1 | 0.2 |
| 2 | 0.2 | 0.3 | 0.1 |

4. If the joint pdf of the random variable is given by f(x,y) = $kxy\,e^{-(x^2+y^2)}$, x > 0 , y > 0 , find the value of k.
5. The tangent of the angle between the lines of regression y on x and x on y is 0,6 and $\sigma_x = \frac{1}{2}\sigma_y$, find the correlation coefficient between X and Y.
6. If the joint pdf of (X,Y) is f(x,y) = $\begin{cases}\frac{1}{4}, 0 \le x < 2\\ 0, otherwise\end{cases}$.

   Find P(x+y ≤1) if P(y=1) = 0.4 and P(Y = 2 ) = 0.6
7. If X and Y have joint pdf f(x,y) = $\begin{cases}x+y, 0<x<1, 0<y<1\\ 0, otherwise\end{cases}$,check whether X and Y are independent.
8. Find the marginal density functions of X and Y if

   f(x,y) = $\begin{cases}\frac{1}{4}(2x+5), 0 \le x \le 1, 0 \le y \le 1\\ 0, otherwise\end{cases}$.
9. Find the marginal density functions of X and Y from the joint density function

   f(x,y) = $\begin{cases}\frac{2}{5}(2x+3y), 0 \le x \le 1, 0 \le y \le 1\\ 0, otherwise\end{cases}$.
10. X and Y are two random variables having the joint density function

    f(x,y) = $\frac{1}{27}(x+2y)$, where x and y assumes the integer values 0,1 and 2. Find the marginal probability distribution of X.
11. Find the value of k if f(x,y) = k(1-x)(1-y) for 0 < x,y < 1is to be a joint density function.
12. Find k if the joint probability density function of a bivariate random variable (X,Y) is

given by f(x,y) = k(1-x)(1-y) if $0 < x < 4$, $1 < y < 5$ and 0 otherwise.

## PART B

1. The joint density finction of a random variable X and Y is f(x,y) = 2, $0 < x < Y < .$ Find marginal and conditional probability density functions. Are X and y independent?

2. Two independent random variables X and Y are defined by $f(x) = \begin{matrix} 4ax, 0 \le x \le 1 \\ 0, otherwise \end{matrix}$ and $f(y) = \begin{matrix} 4by, 0 \le y \le 1 \\ 0, otherwise \end{matrix}$. Show that U = X + Y and V = X +Y are uncorrelated.

3. (X,Y) is a two dimensional random variable uniformly distributed over the triangular region R bounded by y = 0, x = 3 and $y = \frac{4}{3}x$. Find the correlation coefficient $r_{xy}$.

4. X and Y are two random variables having density function $f(x,y) = \frac{1}{8}(6 - x - y)$, $0 < x < 2$, $2 < y < 4$. Find (i) $P( X < 1 \cap Y < 3 )$ (ii) $P( X + Y < 3 )$ (iii) $P( X < 1 / Y < 3 )$.

5. Given the joint distribution of X and Y.

| Y/X | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0.02 | 0.08 | 0.10 |
| 1 | 0.05 | 0.20 | 0.25 |
| 2 | 0.03 | 0.12 | 0.15 |

Obtain (i) marginal distribution and (ii) the conditional distribution of X given Y = 0 .

6. A statistical investigation obtains the following regression equations in a survey. X – Y – 6 = 0 and 0.64X + 4.08 = 0. Here X = age of husband and Y = age of wife. Find (i) Mean of X and Y (ii) Correlation coefficient between X and Y and (iii) $\sigma_y$ = S.D. of Y if $\sigma_x$ = S.D. of X = 4.

7. Given the joint density function $f(x,y) = cx(x-y)$, $0 < x < 2$, $-x < y < x$, Evaluate c . Find the marginal densities of X and Y. Find the conditional density of Y given X = x.

8. Calculate the correlation coefficient for the following heights (in inches ) of fathers (x) And their sons (Y).

| X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
|---|---|---|---|---|---|---|---|---|
| Y | 67 | 68 | 65 | 68 | 72 | 75 | 69 | 71 |

9. If the joint density function of ( X, Y ) is given by $f(x,y) = x + y$ , $0 \le x$ , $y \le 1$, find the correlation coefficient between X and Y.

10. The joint pdf of two random variables X and Y is given by $f(x,y) = \frac{9(1+x+y)}{2(1+x)^4(1+y)^4}$, $0 \le x \le \infty$ , $0 \le y \le \infty$. Find the marginal distribution of X and Y and the conditional distribution of Y for X = x.

11. Two random variables X and Y are related as Y = 4X + 9. Find the coefficient of correlation between X and Y.

12. Marks obtained by 10 students in Mathematics(X) and Statistics (Y) are given below. Find the two regression lines, Also find Y when X = 55.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 60 | 34 | 40 | 50 | 45 | 40 | 22 | 43 | 42 | 64 |
| Y | 75 | 32 | 33 | 40 | 45 | 33 | 12 | 30 | 34 | 51 |

13. If two random variables X and Y have pdf $f(x,y) = k e^{-(2x+y)}$ for x , y > 0 , evaluate k.

14. The joint pmf of the random variables X and Y is $p(x,y) = \frac{e^{-\lambda}\lambda^x p^x q^{x-y}}{y!(x-y)!}$ ,

y = 0,1,2, … x, x = 0,1,2, .. where $\lambda > 0$ , $0 \le P \le 1$, p+q = 1 are constants.Find the marginal and conditional distributions.

15. Two dimentional random variables aX and y have joint pdf f(x,y) = 8xy, $0 < x < y < 1$: 0 otherwise. Find (i) marginal and conditional distributions (ii) Whether X and Y are independent?

16. The joint pdf of the random variable (X,Y) is given by

$$f(x,y) = \begin{cases} kxy, 0 < x < 1, 0 < y < 1 \\ 0, otherwise \end{cases}$$ where

k is a constant. (i) Find the value of k. (ii) Find P(X+Y<1) (iii) Are X and Y independent random variables?

17. If the joint pdf of the random variable (X,Y) is given by

$$f(x,y) = \begin{cases} xe^{-(1+y)}, x > 0, y > 0 \\ 0, otherwise \end{cases}$$ , find f(y/x) and f(Y/X = x).

18. If y = 2x -3 and y = 5x + 7 are the two regression lines, find the values of x and y . Find the correlation coefficient between x and y. Find an estimate of x when y = 1.

19. If the independent random variables X and Y have the variances 36 and 16 respectively , find the correlation coefficient between ( X + Y ) and ( X – Y ).

20. From the following data , find
(i) the two regression equations
(ii) the coefficient of correlation between the marks in Economics and Statistics
(iii)the most likely marks in statistics when marks in Economics are 30

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Marks in Economics | 25 | 28 | 35 | 32 | 31 | 36 | 29 | 38 | 34 | 32 |
| Marks in Statistics | 43 | 46 | 49 | 41 | 36 | 32 | 31 | 30 | 33 | 39 |

21. Two random variables X and Y have joint density function

f(x,y) = $\begin{cases} \frac{xy}{96}, 0 < x < 4, 1 < y < 5 \\ 0, elsewhere \end{cases}$ .Find E(X) , E(Y), E(XY), E( 2X + 3Y ), V(X), V(Y), Cov(X,Y).What can you infer from Cov(X,Y).

22. The joint probability density function of the two dimentional random variable is

f(x,y) = $\begin{cases} \frac{8}{9}xy, 1 < x < y < 2 \\ 0, otherwise \end{cases}$

(i)Find the marginal density functions of X and Y.

(ii)Find the conditional density function of Y given X = x.

23. X is a continuous random variable with pdf given by f(x) = $\begin{cases} kx, 0 \le x \le 2 \\ 2k, 2 \le x \le 4 \\ 6k - kx, 4 \le x \le 6 \\ 0, elsewhere \end{cases}$

Find the value of k and also the cdf 0f f(x).

24. If the joint pdf of a random variable (X,Y) is given by

f(x,y) = $x^2 + \frac{xy}{3}, 0 \le x \le 1, 0 \le y \le 2$ , find the conditional densities of X given Y and Y given X.

25. Let X and Y be random variables having joint density functions

f(x,y) = $\begin{cases} \frac{3}{2}(x^2 + y^2), 0 \le x \le 1, 0 \le y \le 1 \\ 0, elsewhere \end{cases}$ .Find the correlation coefficient $r_{xy}$.