

RANDOM PROCESSES  
QUESTION BANK  
UNIT II  
PART A

- 1) Suppose the duration  $X$  in minutes of long distance calls from your home, follows exponential law with PDF  $f(x) = \begin{cases} (1/8e^{-(x/8)}) & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$ . Find  $P[x > 5]$ ,  $P[3 \leq X \leq 8]$ ,  $P[X \leq 4 / X \geq 4]$ , mean and variance of  $X$ .
- 2) Let  $X$  be a RV with pdf  $f(x) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ . If  $Y = -2 \log X$  find the PDF of  $Y$  and  $E(Y)$ .
- 3) If  $X$  is binomially distributed RV with  $E(X) = 2$  and  $\text{var}(X) = 4/3$ . Find  $P[X = 5]$ .
- 4) A RV is uniformly distributed over  $[0, 2\pi]$ . If  $Y = \cos X$  then
  1. Find PDF of  $Y$  and  $X$
  2.  $E(X)$  and  $E(Y)$
- 5) The life time of a component measured in hours is Weibull distribution with parameter  $\alpha = 0.2$  and  $\beta = 0.5$ . Find the mean life time of the component.
- 6) If  $X$  is a Poisson variate such that  $P[X=2] = 9 P[X=4] + 90 P[X=6]$  find the variance.
- 7) For a BD mean is 6 and SD is  $\sqrt{2}$ . Find the first two terms of the distribution.
- 8) If the RV  $X$  is uniformly distributed over  $(-1, 1)$ . Find the density function of  $Y = \sin(\pi x / 2)$ .
- 9) The life length [in months]  $X$  of an electronic component follows an exponential distribution with parameter  $\lambda = (1/2)$ . What is the probability that the components survives atleast 10 months Given that already it had survived for more than 9 months.
- 10) If  $M$  things are distributed among 'a' men and 'b' women find the number of things received by men is odd.

## PART B

1. Let  $X$  be the length in minutes of a long distance telephone conversation. The PDF of  $X$  is given by  $f(x) = \begin{cases} (1/10)e^{-(x/10)} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$ .
2. The life time  $X$  in hours of a component is modeled by Weibull distribution with  $\beta = 2$  starting with a large number of components. It is observed that 15% of the components that have lasted 90 hours fail before 100 hours. Find the parameter  $\alpha$ .
3. State central limit theorem in Lindberg – Levy form. A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using CLT with what probability can we assert that the mean of the sample will not differ from  $\mu = 60$  by more than 4? Area under the standard normal curve from  $z = 0$  to  $z = 2$  is 0.4772.
4. If the life  $X$  [in years] of a certain type of car has a Weibull distribution with parameter  $\beta = 2$ . Find the value of  $\alpha$  given that the probability that the life of the car exceeds 5 is  $e^{-0.25}$  for these values of  $\alpha$  and  $\beta$  find the mean and variance of  $X$ .
5. It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least exactly and at most 2 defective items in a consignment of 1000 packets using Poisson approximation to Binomial distribution.
6. State and prove the memoryless property of the exponential distribution and geometric distribution.
7. The daily consumption of milk in a city in excess of 20,000 litres is approximately distributed as an gamma variate with the parameters  $k=2$  and  $\lambda = (1/10,000)$ . the city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day?
8. Derive the mgf of negative binomial distribution. Also obtain its mean and SD.
9. Prove that poisson distribution is the limiting case of binomial distribution.
10. Find the MGF of a poisson distribution and hence find its mean and variance.